PHYS302 Fall 2023

Homework 2

3pts

1. (a) Using Maxwell's equations, derive the differential wave equation for the magnetic field of an EM wave. (2 pts)
   (b) Write an expression for the magnetic field which is a solution of (a). Assume the light propagates in the negative y-direction. (1 pt)

4 pts

2. (3.23 from the textbook) Consider a linearly polarized plane electromagnetic wave traveling in the +x-direction in free space having as its plane of vibration the xy-plane. Given that its frequency is 10 MHz and its amplitude is $E_0 = 0.08$ V/m,
   (a) Find the period and wavelength of the wave. (1 pt)
   (b) Write an expression for **E**(x,t) and **B**(x,t). (Use unit vectors.) (2 pts)
   (c) Find the flux density, <S>, of the wave. (1 pt)
   (Note: the "plane of vibration" is the plane that includes both the propagation vector and the electric field.)

3pts

3. (3.26 from the textbook) Pulses of UV lasting 2.00 ns each are emitted from a laser that has a beam of diameter 2.5 mm. Given that each burst carries an energy of 6.0 J,
   (a) determine the length in free space of each wavetrain (i.e. each pulse), and (1.5 pts)
   (b) find the average energy per unit volume for such a pulse. (1.5 pts)

4pts

4. (3.31 from the textbook + an extra part b)
   (a) How many photons per second are emitted from a 100-W yellow lightbulb if we assume negligible thermal losses and a quasi-monochromatic wavelength of 550 nm? (2pts) (In actuality, only about 2.5% of the total dissipated power emerges as visible radiation in an ordinary 100-W incandescent lamp.)
   (b) (not in textbook) Treating the lightbulb like a point source and assuming isotropic radiation (i.e. radiating in a uniform sphere), what is the photon flux density (photons per area per second) 1 m from the light source? (2 pts)

2pts

5. (3.41 from textbook) A surface is place perpendicular to a beam of light of constant irradiance (intensity), I. Suppose that the fraction of the irradiance absorbed by the surface is α. Show that the pressure on the surface is given by

$$Pressure = (2 - \alpha)I/c$$

2pts

6. (3.50 from textbook) Determine the index of refraction of a medium if it is to reduce the speed of light by 10% as compared to its speed in a vacuum.

2 pts

7. (3.67 from textbook) If an ultraviolet photon is to dissociate the oxygen and carbon atoms in the carbon monoxide molecule, it must provide 11 eV of energy. What is the minimum frequency of the appropriate radiation?

Extra credit: 3.66 from the textbook (Note: Cauchy's equation is given in problem 3.63.)

3 extra pts.

HW #2 Solutions

① (a)

$$\vec{\nabla} \times \vec{B} = \mu \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \vec{\nabla} \left(\mu \varepsilon \frac{\partial \vec{E}}{\partial t} \right)$$

→ Use $\vec{\nabla} \times (\vec{\nabla} \times A) = \vec{\nabla} \cdot \vec{A} - \nabla^2 A$

$$\vec{\nabla} \cdot \vec{B} - \nabla^2 \vec{B} = \mu \varepsilon \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

→ Use $\vec{\nabla} \cdot \vec{B} = 0$ & $\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$

$$\boxed{\nabla^2 \vec{B} = \mu \varepsilon \frac{\partial^2}{\partial t^2} \vec{B}}$$

where speed $= v = \frac{1}{\sqrt{\varepsilon \mu}}$

(b) $\vec{B} = B_0 \exp[ik(y + vt)] \, \hat{\imath}$

where $v = \frac{1}{\sqrt{\mu \varepsilon}}$

Also acceptable: sin or cos of $[k(y+vt)]$ and

$\vec{B} \parallel -\hat{\imath}, \hat{k}$, or $-\hat{k}$

2 (3.23)

(a) $f = 10\ \text{MHz}$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8\ \text{m/s}}{10 \times 10^6\ 1/\text{s}} = \boxed{30\ \text{m}}$$

$$T = \frac{1}{f} = \boxed{100\ \text{ns}} \quad (\text{or } 10^{-7}\ \text{s})$$

(b) $\vec{k} \parallel +\hat{x}$ and $\vec{E} \parallel \hat{y}$

Note that $\vec{k} \times \vec{E}$ is in the $\underline{z}$ direction (and parallel to $\vec{B}$).

$$\vec{E}(x,t) = \hat{j}\left(0.08\ \frac{\text{V}}{\text{m}}\right)\cos\left[\frac{2\pi}{30\ \text{m}}\left(x - \underbrace{3 \times 10^8\ \text{m/s}}_{\text{"c" is okay}}\ t\right)\right]$$

$$\vec{B}(x,t) = \hat{k}\,\frac{|\vec{E}|}{c}$$

(c) $$\langle S \rangle = c\,\varepsilon_0\,\frac{E_0^2}{2}$$

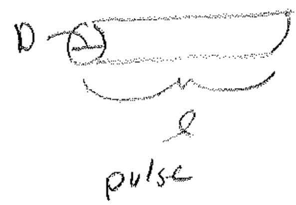
$$\approx 8.5 \times 10^{-6}\ \text{N/m}^2$$

3 (3.26)

(a) $\ell = \Delta t \cdot c = (3\times10^{8}\ m/s)(2.00\,ns)$

$= \boxed{600\ mm}$

(b) Volume is $V = \ell \cdot \pi r^2 = (0.60\,m)(0.00125)^2\pi$



$V = 2.9\times10^{-6}\ m^3$

$$\frac{\text{Energy}}{\text{Volume}} = \frac{(6.0\ J)}{(2.9\times10^{-6}\ m^3)} = \boxed{2.0\times10^{6}\ J/m^3}$$

(4) (3.31 +)

(a) $P = 100\ W = \dfrac{\text{energy}}{\text{second}}$

$E(\text{one photon}) = \dfrac{hc}{\lambda} = 3.6 \times 10^{-19}$ Joules or $2.3\ eV$

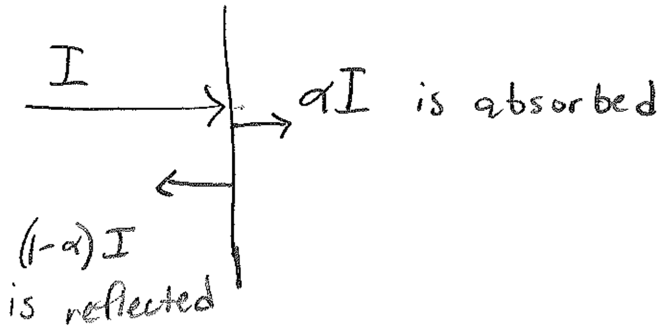
$$\frac{\text{photons}}{\text{second}} = \frac{\text{power}}{(\text{energy}/\text{photon})} = \frac{100\ J/s}{3.6 \times 10^{-19}\ J} = \boxed{2.8 \times 10^{20}\ \text{photons}/s}$$

(b) Surface area of sphere w/ $r = 1\,m$:

$A = 4\pi r^2 \approx 12.6\ m^2$

$$\frac{(2.8 \times 10^{20}\ ph/s)}{12.6\ m^2} = \boxed{2.2 \times 10^{19}\ \frac{ph}{s \cdot m^2}}$$

5 (3.41)



- Pressure from absorbed: $P_{abs} = \dfrac{\alpha I}{c}$

- Pressure from reflected (gets an extra × 2): $P_{refl.} = 2\dfrac{(1-\alpha)I}{c}$

$$P_{total} = P_{abs} + P_{refl} = \frac{I}{c}(2 - 2\alpha + \alpha)$$

$$\boxed{P = \frac{(2-\alpha)I}{c}}$$

6 (3.50)

$$v = \frac{c}{n} = 0.9\,c$$

$$\boxed{n = 1.1}$$

7 (3.67)

$$\mathcal{E} = hf$$

$$f = \frac{(11\,\text{eV})}{(4.14\times10^{-15}\,\text{eV}\cdot\text{s})} = \boxed{2.7\times10^{15}\ \text{Hz}}$$

Extra Credit (3.66)

$$n^2 = 1 + \sum_j \frac{A_j \lambda^2}{\lambda^2 - \lambda_{oj}^2}$$

(First term:) $n^2 \approx 1 + \dfrac{A\lambda^2}{\lambda^2 - \lambda_0^2} = 1 + \dfrac{A}{\left(1 - \frac{\lambda_0^2}{\lambda^2}\right)}$

(Binomial expansion: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$)

$$n^2 \approx 1 + A \cdot (1 + \lambda_0^2/\lambda^2)$$

$$n^2 \approx (1+A)\left[1 + A\lambda_0^2/(1+A)\lambda^2\right]$$

so

$$n \approx \sqrt{1+A}\left[1 + \frac{A\lambda_0^2}{2(1+A)\lambda^2}\right]$$

Cauchy is

$$n = \underset{\uparrow \atop \sqrt{1+A}}{C_1} + \underset{\uparrow \atop \frac{A\lambda_0^2}{2\sqrt{1+A}}}{C_2}/\lambda^2 + \cdots$$